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상호-정보 포텐셜에 기반한 간소화된 블라인드 알고리듬

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Simplified Blind Algorithms based on Cross-Information Potential

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[요 약]

CIP(Cross-Information Potential) 성능 기준을 바탕으로 개발된 블라인드 신호 처리 학습 알고리즘은 충격성 잡음 환경에서도 채널 왜곡으로 인한 심볼 간 간섭에 대한 보상 성능이 뛰어나다. CIP 알고리즘의 단점 중 하나는, 원하는 수준의 확률분포 정확도를 보장하기 위해 N(샘플 사이즈)개의 출력 샘플과 심볼점 간의 모든 상호 작용을 고려하므로 큰 샘플 사이즈가 선호되어 계산 복잡성이 크다는 점이다. 본 논문에서는 다른 모든 샘플보다 현재 샘플이 갖고 있는 정보가 가장 유용하다는 가정하에 모든 출력 샘플 대신 현재 출력 샘플만 상호 작용에 사용하기를 제안하며 이로 인해 제안된 알고리즘의 계산 복잡도가 샘플 크기와 관련이 없게 된다. 시뮬레이션 결과에서 제안한 알고리즘은 N=20에 대해 학습 성능의 큰 손실 없이 계산 복잡도를 약 21배 정도 감소시켰으며, 이는 제안한 성능 기준 및 알고리즘이 기존 CIP 알고리즘보다 실제 구현에 더 적합할 수 있음을 나타낸다.

[Abstract]

The learning algorithm for blind signal processing developed from the performance criterion of cross-information potential (CIP) has superior compensation performance for intersymbol interference induced by channel distortion, even under impulsive noise. One of the drawbacks of the CIP algorithm is a heavy computational complexity caused by considering all the interactions between N (sample size) output samples and the symbol points where a large sample size is preferable to guarantee a desired level of accuracy in distribution estimation. In this paper, the idea of taking only the current output sample into their interactions instead of all output samples is proposed under the assumption that the information the current sample has is the most useful of all other samples; this leads to the computational complexity of the proposed algorithm not being related to the sample size. The simulation results show that the proposed algorithm significantly reduces the computational complexity by approximately 21 times for N=20 without noticeable loss of learning performance, which indicates that the proposed criterion and algorithm are more suitable for practical implementations than the conventional CIP algorithm.

색인어 : 상호-정보 포텐셜, 계산 복잡성, 블라인드 신호 처리, 단순화, 충격성 잡음

Keyword : Cross-Information Potential, Computational Burden, Blind Signal Processing, Simplification, Impulsive Noise

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Ⅰ. Introduction

Blind algorithms for signal processing or communication systems are being effectively utilized in overcoming signal distortion or inter-symbol interference induced by storage media or communication channels. Blind algorithms do not need a training sequence to recover the distorted signal or to restart after a communications breakdown[1],[2]. This advantage has been increasingly utilized in computer communication networks and broadcasting systems[3].

The training of blind adaptive systems has usually been accomplished by using the constant modulus error (CME) and the mean squared error (MSE) criterion[4],[5]. Unlike the MSE criterion that utilizes CME energy, ITL methods introduced by Princepe uses a nonparametric probability density function (PDF) estimation and information potential (IP) which measures interactions among pairs of data samples[6],[7]. It contains higher order moments of the PDF and is much simpler to estimate directly from samples than conventional moments expansions.

The MSE based learning algorithms are known not to yield sufficient performance in non-Gaussian noise, such as impulsive noise environments[6],[8]. On the other hand, learning algorithms developed from the ITL methods can break through these obstacles partly due to the kernel-based PDF estimation and information potential[6]. The minimum error entropy (MEE) criterion as one of ITL criteria is a powerful approach for non-Gaussian signal processing as well as for robust machine learning[9],[10]. One of its drawbacks is that it has been designed only for supervised learning. Another problem is its computational complexity including double summation operations. Recently a simplified version of MEE has been introduced by utilizing only two error samples that contribute mostly to performance enhancement[11].

Besides the concept of IP, through the extension of the definition of correlation function for random processes, the generalized correlation function called correntropy has become another performance criterion of ITL and developed to be applied to blind signal processing[12]. The correntropy is related to the probability of how similar two random variables are in the neighborhood of the joint space[13].

In this paper, in order to develop efficient and unsupervised learning algorithms which are robust in impulsive noise environment, we briefly explain blind algorithms based on correntropy and IP, and then propose a simplified IP criterion and its related algorithms for computational complexity reduction without noticeable loss of performance.

Ⅱ. Blind Performance Criteria based on Gaussian Kernel

With a kernel size σ, the Gaussian kernel is defined as

$$G_\sigma(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{-x^2}{2\sigma^2}\right) \tag{1}$$

Given N(sample size) data samples $\{x_i\}$, the correntropy function V_X [n] with sample distance or time distance n is described as follows[12],[13].

$$V_X[n] = \frac{1}{N-n+1}\Sigma_{k=n}^{N} G_\sigma(x_k - x_{k-n}) \tag{2}$$

When it comes to dissimilarity measure as a kind of error concept, the squared difference C_{SY} between the transmitter correntropy $V_S[n]$ and that of the receiver $V_Y[n]$ can become a cost function or a performance criterion as proposed in[12]. We refer to this C_{SY} as MSCD (mean squared correntropy distance) in this paper for convenience's sake.

$$C_{SY} = \frac{1}{N}\Sigma_{n=1}^{N}(V_S[n] - V_Y[n])^2 \tag{3}$$

On the other hand, the information potential (IP) can be a kind of performance criterion. The concept of information potential may be summarized as follows.

When we see the value of a given data sample x_i as the location of the data axis x, the kernel function $G_{\sigma\sqrt{2}}(x_j - x_i)$ for two data samples x_i and x_j on the axis produces exponential decaying outcomes regarding the distance between x_i and x_j. This can be interpreted as the Gaussian kernel $G_{\sigma\sqrt{2}}(x_j - x_i)$ plays a role as a potential field that induces the interactions between the two particle-like data samples x_i and x_j. The perspective regarding a data sample as a particle with information in an information potential field becomes the basis of the concept of information theoretic learning (ITL)[6],[7].

Then $\Sigma_{j=1}^{N} G_{\sigma\sqrt{2}}(x_j - x_i)$ is corresponding to the summed interactions towards to x_i and $\frac{1}{N^2}\Sigma_{i=1}^{N}\Sigma_{j=1}^{N} G_{\sigma\sqrt{2}}(x_j - x_i)$ becomes the averaged total interaction among all data samples on the x axis. This function of total interactions is called information potential[6]. That is, the information potential IP_{XX} for a given set of samples $\{x_1, x_2, \cdots, x_N\}$ can be written as

$$IP_{XX} = \frac{1}{N^2}\Sigma_{i=1}^{N}\Sigma_{j=1}^{N} G_{\sigma\sqrt{2}}(x_j - x_i) \tag{4}$$

We may notice in (4) that when the samples are placed close to each other the potential energy becomes high and vice versa.

The information potential can be expressed with a probability density function $f_X(x)$ based on kernel density estimation method[14].

$$f_X(x) = \frac{1}{N}\Sigma_{i=1}^{N} G_{\sigma}(x - x_i) \tag{5}$$

Then, $\int f_X^2(x)dx$ becomes

$$\int f_X^2(x)dx = \frac{1}{N^2}\int \Sigma_{i=1}^{N} G_{\sigma}(x - x_i)\Sigma_{j=1}^{N} G_{\sigma}(x - x_j)dx$$
$$= \frac{1}{N^2}\Sigma_{i=1}^{N}\Sigma_{j=1}^{N} G_{\sigma\sqrt{2}}(x_i - x_j) \tag{6}$$

Therefore,

$$IP_{XX} = \int f_X^2(x)dx \tag{7}$$

For the two different densities $f_S(s)$ and $f_Y(y)$, the cross information potential (CIP) called information potential CIP_{SY} is defined in[7] as

$$CIP_{SY} = \int f_S(\alpha) f_Y(\alpha) d\alpha \tag{8}$$

We can notice that when the samples s_i and y_i are placed close together the information potential CIP_{SY} becomes high and vice versa.

For application to blind equalization in communication with $M-ary$ modulation schemes, technology, the density $f_S(s)$ can be constructed from the transmitter symbol set $S = \{A_1, A_2, \cdots, A_M\}$ and density $f_Y(y)$ is from the receiver output samples $Y = \{y_k, y_{k-1}, \cdots, y_{k-N+1}\}$. When the transmitted symbol at time k is assumed to be randomly chosen with equal probability, $f_S(s)$ and $f_Y(y)$ can be obtained

$$f_S(s) = \frac{1}{M}[\delta(s - A_1) + \delta(s - A_2) + \ldots + \delta(s - A_M)] \tag{9}$$

where $\delta(s)$ is the Dirac-delta function on the s axis.

And through the kernel density estimation method with the sample size N as in (5) $f_Y(y)$ becomes

$$f_Y(y) = \frac{1}{N}\Sigma_{i=k-N+1}^{k} G_{\sigma}(y - y_i) \tag{10}$$

Then the equation (8) using (9) and (10) can be rewritten as

$$CIP_{SY} = \frac{1}{M}\frac{1}{N}\Sigma_{m=1}^{M}\Sigma_{i=k-N+1}^{k} G_{\sigma}(A_m - y_i) \tag{11}$$

The two ITL-type criteria, C_{SY} of MSCD, and CIP_{SY} of CIP can be summarized as in (3) and (11), respectively.

For comparison's sake, the common blind criterion P_{CMA} for the constant modulus algorithm (CMA) can be rewritten as

$$P_{CMA} = E[(|y_k|^2 - R_2)^2] \tag{12}$$

where $R_2 = E[|s_i|^4]/E[|s_i|^2]$

Minimizing this criterion leads to force equalizer output powers to have the same value, R_2. In $M-ary$ modulation schemes, the power of each desired signal has different values. The force induced from minimizing P_{CMA} will lose its target direction because the cost function forces the equalizer outputs to obtain the same output power R_2 in spite of each symbol's desired power being different from each other. This may lead the CMA cost function R_2 to ill-convergence more severely in impulsive noise situations.

Ⅲ. Proposed CIP Criterion

The fact that the information potential IP_{XX} in (2) is the interaction energy among all data samples on the x

axis leads us to view the CIP in (11) as the interactions between the M symbol points and N output samples $\{y_k, y_{k-1}, \cdots, y_{k-N+1}\}$.

Instead of considering all the interactions of N output samples towards to the symbol points, we may consider only that of y_k under the assumption that the information the current sample y_k has is the most useful of all other samples. This assumption will be verified through experiment in Section V.

So, we intend to consider only the interactions between y_k and all symbol points $\{A_1, A_2, \cdots, A_M\}$. Then the simplified CIP becomes

$$SCIP_{SY} = \frac{1}{M} \Sigma_{m=1}^{M} G_\sigma(A_m - y_k) \tag{13}$$

This proposed (13) will be referred to as SCIP (simplified CIP) in this paper. It may be needed to observe performance difference between CIP_{SY} and its simplified version $SCIP_{SY}$ through the experiment in Section V.

Ⅳ. Blind Learning Algorithms

For the input vector $X_k = [x_k, x_{k-1}, x_{k-2}, \cdots, x_{k-L+1}]^T$ and weight $W_k^T = [w_{k,0}, w_{k,1}, w_{k,2}, \cdots, w_{k,L-1}]$ at time k, the linear combiner produces the output $y_k = W_k^T X_k$. The MSCD algorithm for weight update is obtained by minimizing C_{SY} in (3) with a step size μ_{MSCD}.

$$W_{k+1} = W_k - \frac{\mu_{MSCD}}{(N-n+1)\sigma^2} \Sigma_{n=1}^{M} \Sigma_{i=k-N+n}^{k} (V_S[n] - V_Y[n]) G_\sigma(y_i - y_{i-n})(y_i - y_{i-n})(X_i - X_{i-n}) \tag{14}$$

The CIP algorithm is obtained by maximizing CIP_{SY} in (11) as

$$W_{k+1} = W_k + \mu_{CIP} \frac{2}{MN\sigma^2} \Sigma_{i=k-N+1}^{k} \Sigma_{m=1}^{M} (A_m - y_i) \cdot G_\sigma(A_m - y_i) \cdot X_i \tag{15}$$

Likewise, maximizing $SCIP_{SY}$ leads to SCIP algorithm with a step size μ_{SCIP} as

$$W_{k+1} = W_k + \mu_{SCIP} \frac{2}{M\sigma^2} \sum_{m=1}^{M} (A_m - y_k) \cdot G_\sigma(A_m - y_k) \cdot X_k \tag{16}$$

For comparison, the CMA algorithm (CMA) obtained from minimizing P_{CMA} with respect to system weights can be written as

$$W_{k+1} = W_k - \mu_{CMA} 2y_k \cdot (|y_k|^2 - R_2) \cdot X_k \tag{17}$$

It may be worthwhile to observe different performance among CMA in (12), MSCD in (3), CIP in (11) and its simplified version SCIP in (13) through the experiment in Section V.

Ⅴ. Simulation Results and Discussion

In this Section, the learning performance of the MSCD algorithm and its simplified version SCIP is analyzed in the experiment of a base-band communication system as shown in Fig. 1.

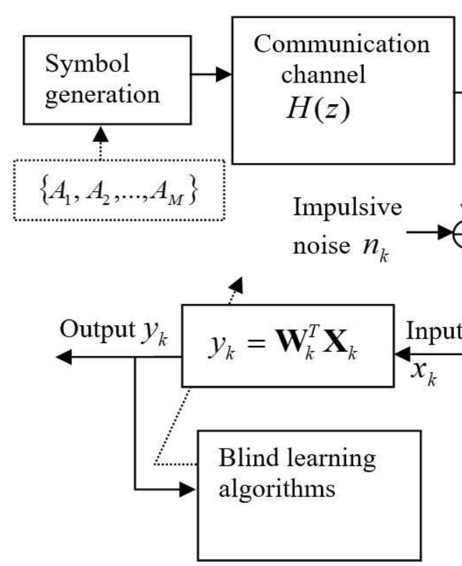


Fig. 1. A Base-band communication system for the experiment

For the data generation of 4 symbols ($M=4$), one of $\{A_1=1, A_2=-1, A_3=3, A_4=-3\}$ is randomly chosen (equiprobable) and sent at time k. The transmitted symbol is through the communication channel $H(z)$ where the symbol is distorted by intersymbol interference and then corrupted by impulsive noise.

The receiver is equipped with a linear combiner $y_k = W_k^T X_k$ with $L = 11$. The transfer function $H(z)$ is in (18) as in[8].

$$H(z) = 0.304 + 0.903z^{-1} + 0.304z^{-2} \tag{18}$$

The impulse response of (18) corresponds to

$$h(t) = 0.304\delta(t) + 0.903\delta(t-T) + 0.304\delta(t-2T) \tag{19}$$

where T is the symbol period.

The impulsive noise comprises impulses and white Gaussian noise. The impulses are generated as in[12],[13] by Poisson process with variance 50 and occurrence rate 0.03. The variance of the Gaussian noise is set 0.001. A sample of the impulsive noise is shown in Fig. 2.

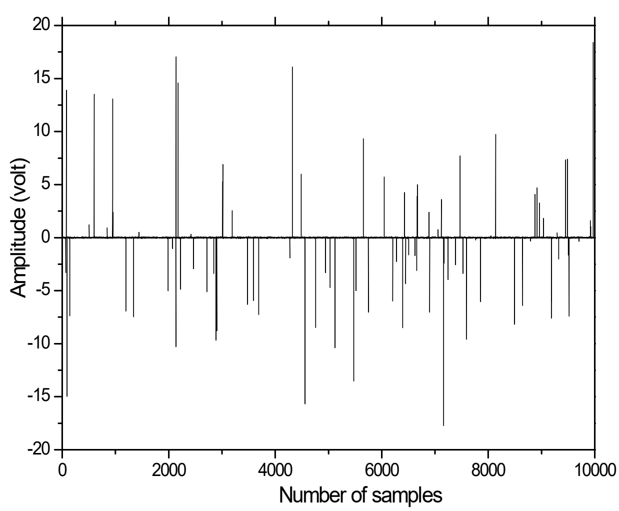


Fig. 2. A sample of impulsive noise (a sample of random impulsive noise generated by the method[12])

The system weights of the CIP and SCIP are updated with the common step size $\mu_{CIP=}\mu_{SCIP} = 0.005$ and the kernel size $\sigma = 0.6$. The step sizes for CMA and MSCD, μ_{CMA} and μ_{MSCD} are set 0.01 and 0.00001, respectively. The kernel size for MSCD is 2.8 and the sample size $N = 20$.

In Fig. 3 showing the MSE learning curves we may observe that the learning curve of CMA in (3) does not converge below −7 dB which indicates that CMA is inappropriate. But the CIP or SCIP algorithms show fast and stable convergence to around −27 dB. The MSE learning curves of the proposed SCIP and the conventional CIP result in similar performance with only slightly different steady state MSE of −2.7 dB and −2.75 dB, respectively. That difference can be viewed as negligible in most communication systems which usually demand performance difference of above 3 dB in order for a new system to be judged as better or superior.

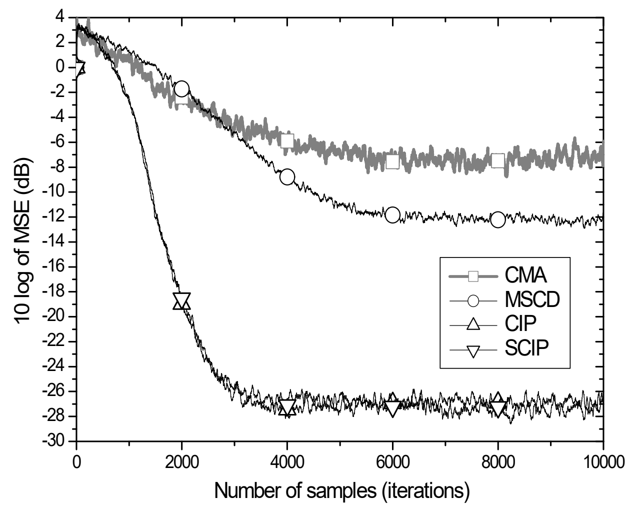


Fig. 3. MSE learning curves

We may see that this property of performance equality that the SCIP has reveals that the interactions between y_k and symbol points $\{A_1, A_2, \cdots, A_M\}$ are enough for the weight update to count in rather than considering all the interactions of $\{y_k, y_{k-1}, \cdots, y_{k-N+1}\}$ and the symbol points $\{A_1, A_2, \cdots, A_M\}$.

Besides this property, the proposed SCIP algorithm has a figure of merit that its computational complexity in multiplication is remarkably reduced. For the sake of convenience of comparison, $2/\sigma^2$ and the Gaussian kernel $G_\sigma(A_m - y_i)$ which commonly exist in both methods, CIP in (10) and SCIP in (11), are treated as constants.

The block-processing method (10) demands $3MN$ multiplications at each iteration time while the proposed method (11) requires $3M$ multiplications. It is important that the computational complexity of the proposed one is not related with the sample size since a large sample size is preferable in order to guarantee a desired level of accuracy[15].

Fig. 4 shows the number of multiplications with respect to sample size and it is apparent that the proposed SCIP algorithm is more appropriate to practical implementations.

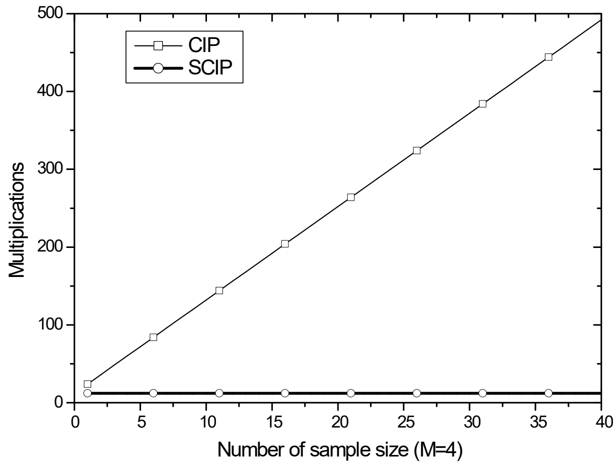


Fig. 4. Number of multiplications required for CIP and SCIP with respect to the sample size N

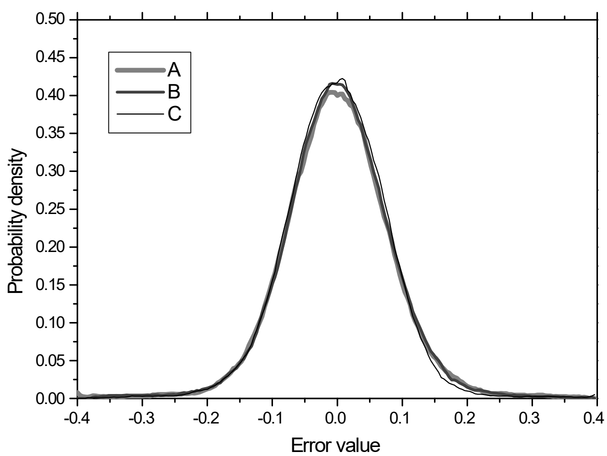
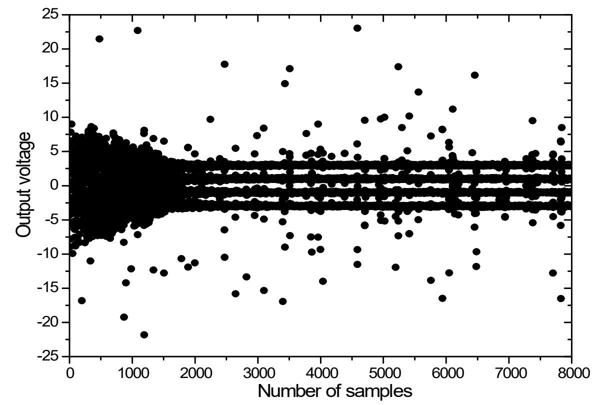
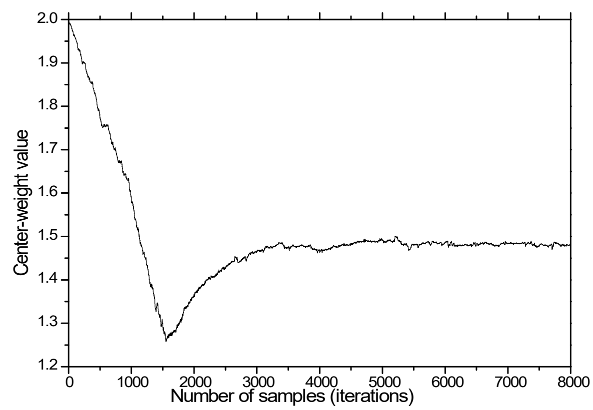


Fig. 5. System error distribution for several interactions (A: between y_k and the symbol points, B: between y_k, y_{k-1} and the symbol points, C: between y_k, y_{k-1}, y_{k-2} and the symbol points)

On the other hand, the assumption that the current output y_k has more useful information than all other samples $\{y_{k-1},...,y_{k-N+1}\}$ can be verified through the performance comparison of the system error distribution which depicted in Fig. 5. In Fig. 5, the error distribution of case A which considers only the interactions between the current output y_k and the symbol points $\{A_1,A_2,...,A_M\}$ has a bell shape distribution narrow enough to gather most error samples near zero. The contribution to error performance enhancement by adding the interactions between the past output y_{k-1} and the symbol points is insignificant as we can see in the case B. We can also observe the similar results in case C where the interactions between y_{k-1}, y_{k-2} and the symbol points are added. This indicates that the information the current output sample y_k has is the most useful of all other outputs.



(a)



(b)

Fig. 6. Output signal (a) and center-weight trace (b)

It is noticeable how the impulsive noise is processed in the learning algorithm SCIP. The resulting output signal in Fig. 6(a) shows that the output samples gather on their corresponding target symbol points after convergence though the impulses are not removed. The weight trace (the center weight is chosen for convenience's sake) in (b) verifies the robustness of SCIP by showing that it converges its steady state value without any disturbance or fluctuations even under the strong impulsive noise.

Ⅵ. Conclusion

For overcoming the multipath channel distortions and non-Gaussian noise effects, many performance criteria and related blind algorithms have been developed based on the information potential concept which was built onprobability estimation. Despite its superior performance in non-Gaussian impulsive noise, the algorithms require heavy computational burden due to a large sample size for guaranteeing a desired level of accuracy of probability estimation. The proposed performance criterion and weight update algorithm significantly reduces the computational complexity without noticeable loss of performance in unsupervised learning and impulsive noise situations. This indicates that the proposed algorithm can replace the supervised MEE algorithms for unsupervised learning not needing training data and the CIP algorithm for reduced complexity. We may conclude thatthe proposed SCIP algorithm can be more appropriate to practical implementations than the conventional CIP algorithm.

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